7.8 Material Implication

7.8.1

Definition: A truth-functional compound statement, consisting of two components. The "truth-functor" is "if….then…", where the ellipses are what contain the simpler component statements. This kind of truth-functional statement is also known as a *hypothetical* statement or "implication".

The part of the truth-functional statement that comes after "if" is the "antecedent", or "hypothesis", and the part coming after "then" is the "consequent".

Denotation is "$\rightarrow$"

Usages:

If P, then Q;

P is true only if Q is true

P implies Q

Q is true whenever P is true.

For P to be true, it is necessary for Q to be true.

For Q to be true, it is sufficient for P to be true.

Example:

If it is the first Sunday, then we are having apologetics class.

So if we let *p* = "it is the first Sunday" and we let *q*= "we are having apologetics class", we have:

$p \rightarrow q$

Now let's think about this for a moment:

Before the first Sunday rolls around, suppose I told you "If it is the first Sunday, then we are having apologetics class." Our previous study tells us there are 4 possible scenarios that could happen here: and they are:

1. It is the first Sunday, and we are having apologetics class.
2. It is the first Sunday, and we are **not** having apologetics class.
3. It is **not** the first Sunday, and we are having apologetics class.
4. It is **not** the first Sunday, and we are **not** having apologetics class.

Which of the above scenarios would support my assertion, "If it is the first Sunday, then we are having apologetics class."? Which scenario(s) would contradict my assertion?

If the first scenario occurs, it totally supports my assertion.

If the second scenario occurs, it contradicts what I asserted in that I said, If 'p' occurs, then 'q' must occur. Well, the scenario 2 says, 'p' is occurring, but 'q' is not occurring; hence, the contradiction.

If the third or fourth scenarios occur, you can't accuse me of lying, because I never said anything about what occurs on non-first Sundays (at least in this example lol). Hence, my claim is only to be evaluated on the content it affirms, not on what it does not affirm.

The truth table for a conditional is as follows:

P	Q	p →q
T	T	T
T	F	F
F	T	T
F	F	T

Table 1.

*you may be asking "why is the truth-functional statement 'If it is the first Sunday, then we are having apologetics class.'considered true if *p* is false and *q* is true, and if both *p & q* are false?" A brief answer is that if the truth-functional statement were considered to be 'true' when p=F and q=T,and when p=F and p=F, then the truth table for p→q would be identical to p^q, which would cause some 'stickiness' …also, we since we said the antecedents were false, this prevents us from actually using the conditional to infer the consequent. So we refer to the truth-functional statement as being 'vacuously true' in such scenarios. See http://en.wikipedia.org/wiki/Vacuous_truth for some interesting information on this.

It may be a good idea to understand "p→q" in terms of sufficient conditions and necessary conditions. They are defined as follows:

Sufficient condition: one in whose presence, the consequent phenomenon must happen.

Necessary condition:, one in whose absence, the consequent phenomenon cannot occur.

P is defined as a sufficient condition for *q*, in that when *p* is present, *q* then must occur..

Q is defined as a necessary condition for *p* in that whenever *q* is absent, it is implied that *p* is then absent.

P	Q	p →q	Relationship to sufficient & necessary conditions
T	T	T	*The conditional truth value is true, because in the presence of p (ie, p=true), q is also true, making the truth-functional statement true.*
T	F	F	The conditional truth value is false because 1.) our sufficient condition, p, exists, but q does not. And 2.) our necessary condition is truly absent or false, and we expect that p will absent or false, but p is in fact true---thus, the conditional is false in this scenario.
F	T	T	The conditional truth value is true because our sufficient condition is not present/true, thus impeding us from actually using the rule of the conditional. Also, our necessary condition is present, thus disabling us from making any implication via the rules of conditionals.
F	F	T	q is the necessary condition for p. Since q is absent (F), p is absent (F).

Table 2

So, hearkening back to the Lecture 1 Analysis document, I gave you a “cliffhanger” when we came across an example

Example 3.	In symbols:
Premise (1) If it rains outside, then the grass will be wet.	(denoted p→q)
<u>Premise (2) the grass is, indeed, wet.</u>	<u>q</u>
Therefore, it rained outside	therefore p

This above example 3 was an attempt to take a standard conditional of the form “p→q”, and then to affirm “q”, and to then imply “p”. This was the fallacy of “*Affirming the Consequent*” (very common in science). I said I would explain why “p→q” is not equivalent to “q→p” (called the ‘converse’)….So first, we’ll use intuition & basic language (from our example 3 here) :

p→q =“If it rains outside, then the grass is then wet.”

(i.e., so based on this assertion, you can bank on the fact that when it rains, it gets wet).

q→p =”If the grass is then wet, then it is raining outside.”

(i.e., paraphrased: *Whenever* the grass is wet, it had to be raining outside. I think we can see the flaw in this Reasoning, in that we can think of at least 1 time where the grass was wet, and it was not Raining outside: dew, hose, sprinklers, water line break.)

Let’s look at and compare the truth tables for p→q and q→p.

P	Q	p →q	q→p
T	T	T	T
T	F	***F***	***T***
F	T	***T***	***F***
F	F	T	T

Table 3

Note that the truth table values for a conditional and its converse are not consistent (see bold italicized values).

Note: this concept of finding the “contradictory scenario” (where if we have ‘p→q’ and assuming p=true and q=false) will play an important part when we develop some basic proofs.

Just a quick glimpse of how this is applied is an example of a *Contradiction* proof, which is very effective. When the proponent argues If A then B, (if A→B is not true) we can disprove A→B by *assuming* A and ~B (which is the contradiction of A→B), and through an undesignated number of logical steps arrive at a conclusion of “R & ~ R”---a contradiction which violates one of the most fundamental laws of thought. We shall explore this in more detail (and some very biblical/relevant & concrete examples shortly). There’s a bit more to be said about conditionals though before getting there…And we shall see that when used correctly, *assumption* is not such a bad idea, compared to when it’s used incorrectly.

7.8.2

Another commonly committed fallacy concerning conditionals is called “*Denying the Antecedent”*, and as the name suggests, it takes on the following form in an argument:

Premise 1: A→B

Premise 2: ~A

Conclusion: ~B

For example, in Revelation 3:5, it says “He who overcomes shall be clothed in white garments, and I will not blot out his name from the Book of Life;…”(NKJV). Certainly the local context (and global context of the whole scripture) is key in interpreting this verse; however, those of a particular school of thought not only disregard context, but they commit the fallacy of “Denying the Antecedent” here. They argue fallaciously as follows:

Premise 1: If you overcome, then you will be clothed in white garments and Christ will not blot out your name from the Book of life.

<u>Premise 2: You don't overcome</u>

Conclusion: Christ will blot out your name from the book of life. (i.e., forfeiting salvation). The proponent of this argument invalidly concludes that Christ will do something, based on an invalid implication.

Now, why is it that A→B is not equivalent to ~A→~B?

Well, first, using intuition, to say that "if A, then B", is just that---if A is true, then B is true… When you're given a conditional, you can make a limited amount of deductions about it. One is modus ponens:

Premise 1: A→B

<u>Premise 2: A</u>

Conclusion: B

Now using some more intuition we know that if we're told that B occurs if A occurs, we cannot (based only on the given info that B occurs if A occurs) conclude that B *only occurs* if A occurs, and that there isn't some "C" out there that should it occur, then B occurs as well. Nor can we say that if A, then only B occurs. (*Note, the "C" for our Rev 3:5 example above is, in fact "~A" or not overcoming). The truth table compares A→B with ~A→~B;

A	B	A→B	~A	~B	~A→~B
T	T	T	F	F	T
T	F	***F***	F	T	***T***
F	T	***T***	T	F	***F***
F	F	T	T	T	T

Table 4

We're interested in the fact that there is variation in the truth-values (bold and italicized). This shows the statements A→B and ~A→~B are not logically equivalent.

Think about the following:

Are these sound arguments?: If it's the first Sunday, we'll have apologetics class;

<u>It's not the first Sunday:</u>

Conclusion: We'll not have apologetics class;

If it rains outside, the grass will be wet;

<u>It is not raining outside;</u>

Conclusion: the grass will not be wet;

If you overcome, you name will not be blotted out of the Book of Life:

<u>You do not overcome;</u>

Your name will be blotted out of the Book of Life.

7.8.3 *Modus Tollens (the mode that denies by denying)*

Another valid deduction we can make from the conditional is Modus Tollens (*meaning 'mode that denies by denying')*, which is based on the fact that a conditional truth-functional statement is such that *A* cannot be true and *B* false (i.e, ~(A*~B)), is as follows:

Premise 1: A→B

<u>Premise 2: ~B</u>

Conclusion: ~A

Note: (so this just says, 'if not B, then that implies not A….for we never want B to be false (in premise 2) and A (Conclusion) to be true, or else we've contradicted premise 1, which says when A is true, B must be true. This can also be seen more easily in the truth table for the conditional where the contradiction of a condition is where the hypothesis is true, and the consequent is false.

7.9

(Up next…the biconditional)

Biconditionals:. Denoted A< -- > B

Note; in the basic conditional A→B, if you were given "A", you could imply that "B" occurs/is true as well. However, if B occurs/is true, we could not necessarily imply that A occurred/is true, but it possible that some "C" was the condition for that "B" to occur. So, in the conditional A was a sufficient, but not necessary condition for B, and B was a necessary, but not sufficient condition for A. However, the biconditional is said as follows:…

A if and only if B, which can be broken down as:

(A if B) AND (A only if B), which can be broken down as:

B→A AND A→B, …put into a truth table as:

A	B	A→B	B→A	(B→A)*(A→B)
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Table 5

We see that the biconditional statement is only true whenever both, A and B are true, or when both are false. So, A is a sufficient condition of B, in that when A is true, B must be true for the biconditional to hold, and likewise, B is a sufficient condition of A such that when B is true, A must be true for the biconditional to hold. Furthermore, A is a necessary condition for B in that when A is absent/ false, B must be absent or false for the biconditional to hold, and B is a necessary condition for A in that when B is absent, A must be absent/false for the biconditional to hold.

In other words, both A and B must be true or false. Never alternating…or the biconditional is false. So, both A & B must agree in truth-value for the truth-functional statement to be true.

For example:

<u>You receive eternal life if and only if you've believed the gospel.</u> (as compared to *you receive eternal life if you believe the gospel, --said another way---you believe the gospel only if you receive eternal life).*

The biconditional " You receive eternal life if and only if you believe the gospel" cannot be true if it is found that one believes the gospel and does not receive eternal life, or if one receives eternal life and hasn't believed the gospel.

Another example: Corydon and I will both be present to teach apologetics if and only if it is the first Sunday of a month. This is saying that if it is a first Sunday of a month, then Corydon & I will team teach, AND if Corydon and I team teach, then it is the first Sunday of the month.---meaning, when "team teaching" is true, then this implies that "1st Sunday of the month" is also occurring, AND if first Sunday of the month is true, then "team teaching" is also true. Furthermore, if "team teaching" is not occurring, then you can validly conclude that it is not the first Sunday of the month, and if 1st Sunday of the month is not occurring, then you can validly conclude that team teaching isn't occurring.

Other valid deductions/implications of the biconditional:

In A< -- >B , we see that if A is not true, that this automatically implies that B is not true as well (were as this is not the case in A→B). Likewise, if B is not true, then A is not true. So ~A →~B, and ~B→~A are true of biconditionals.

7.11 Testing Arguments with truth tables:

Since valid arguments cannot contain true premises and false conclusions, we can determine if an argument is valid by identifying if such is the case; for example:

If Bruce is an athlete, he has good lungs.

He has good lungs.

Conclusion: Bruce is an athlete.

P	Q	p→q (premise 1)	Q (premise 2)	P (conclusion)
T	T	T	T	T
T	F	F	F	T
F	T	***T***	***T***	***F***
F	F	T	F	F
Step 1	step 1	Step3	step 2	step 2

(fill in the columns in the designated order)

Table 6.

Don't be duped into seeing the string of "T" values on top, and then think this is a valid argument; for both valid and invalid arguments can have true premises and true conclusions (as this example demonstrates an invalid argument which has true premises and a true conclusion).

Here we see that (bold/italicized) show that when the premises are true, we get a false conclusion, thus showing the argument could not possibly be valid, since valid (i.e., logically structured) deductive arguments must yield a true conclusion when the premises are true.

We know that this invalid argument form is that fallacy of *affirming the consequent.*

Now for another example:

If Jesus raised Himself from the dead, then He must be the Son of God.

Jesus raised Himself from the dead.

Conclusion: He is the Son of God.

P	Q	p→q (premise 1)	P (premise 2)	Q (conclusion)
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

Table 7.

Here, we have an example of modus ponens, and there is no possibility here for all the premises to be true, and the conclusion false.

Proofs:

Now, using all this theory, we will implement three methods of proof to prove P→Q:

1.) Direct Method: Assumes P, …(logical sequence of steps)…Concludes Q

2.) Contrapositive Method: Assumes ~Q…(logical sequence of steps)…Concludes ~P

3.) Contradiction Method: Assumes P and ~Q…(logical sequence of steps)…Conclude R & ~R

   i. Note: we don't know when 'R' will come about in this logical sequence.

**claims/potential Arguments to Apply this theory on:

Cosmological Argument for the existence of God.

Since God can do all things, can he make a rock so big that He cannot lift? (and it's associated reasoning)

Other gods are real yet the God of the Bible was recognized as the Top God over them all. (and its associated reasoning)

If God exists, then evil would not be present in the world.

Is it that God does something because it is good, or is it good because He said so?

If Moses was the most humble man on the earth at the time, why would he make such an immodest evaluation by calling himself humble in the writing.